Chainlets in Electromagnetism: A tentative, yet enthusiastic survey.

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1 Outline

It is in continuation of a longstanding tradition, that the electromagnetic theory lends itself as a test environment for the application of modern geometrical techniques. Chainlet geometry is an exciting new topic on the horizon of geometrical integration theory [1, 2]. It constitutes a framework that bears within the hope for a true unification of discrete and continuous approaches to physical problems.

Conservative tentatives to incorporate aspects of chainlet geometry in electromagnetism use the geometrical limit processes in Banach spaces only for the modeling of spacial entities. Faraday- and Ampère-Maxwell fields are modeled by even and odd differential forms, respectively. The double-duals, i.e., de Rham currents, account for singular distributions of charges or currents.

It is my conviction that a more radical approach is in order to relish the full power of Jenny Harrison's discoveries. I will show that the entire Ampère-Maxwell complex is naturally modeled in the space of chainlets, whereas the Faraday complex is best modeled in the dual space. The double-dual de Rham currents are not needed. Similar approaches, using chains and forms, have been highlighted in [3] and [4].

- Familiar objects of electromagnetism, such as current filaments, current- and charge densities, double-layers, and so forth, find their natural place in the framework of chainlet geometry.
- Energy-functionals arise naturally from the duality of Faraday and Ampère-Maxwell fields, and material laws can be introduced by the Legendre Transform [5].
- Odd differential forms become a natural supplement for a technical treatment of boundary-value problems.
- The gap between network theories, that model Ampère-Maxwell fields by chains, and the continuous theory is bridged.
- All discrete operators that figure in a discrete formulation including a boundary term are restored their intuitive meaning.
- The resulting theory of electromagnetism resembles closely what de Rham had outlined in a first draft of his theory of currents [6], well before he defined them as functionals on the space of differential forms [7].

Much remains to be done before a chainlet theory of electromagnetism will affect our computational practice. On the long run, however, I hope that the electromagnetic theory will lead the way for other phyiscal theories to adopt a framework that combines, in an unprecedented way, intuition with mathematical rigor.

References

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