hp-BEM for Electromagnetic Scattering

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The mathematical problem we are going to look at is as follows

$$\begin{aligned} \frac{1}{k}\nabla \times \nabla \times \boldsymbol{u} - k\boldsymbol{u} &= \boldsymbol{0} && \text{in } \Omega^+\,, \\ \gamma_D^+ \boldsymbol{u} &= \boldsymbol{m} && \text{on } \Gamma\,, \\ \left|\nabla \times \boldsymbol{u}(\boldsymbol{x}) \times \frac{\boldsymbol{x}}{|\boldsymbol{x}|} - ik\boldsymbol{u}(\boldsymbol{x})\right| &= o\big(\frac{1}{|\boldsymbol{x}|^2}\big), && |\boldsymbol{x}| \to \infty\,. \end{aligned} \tag{1}$$

Its solution describes the electric field component of an scattered electromagnetic wave propagating in the unbounded domain Ω^+. The existence of a fundamental solution for the differential operator

$$\frac{1}{k}\nabla \times \nabla \times -k\,, \quad k > 0$$

yields a representation for $\boldsymbol{u}$ in Ω^+. The so-called representation formula describes the electric field $\boldsymbol{u}$ as an extension of its Dirichlet data $\gamma_D^+\boldsymbol{u}$ and its Neumann data $\gamma_N^+\boldsymbol{u}$. The Neumann data $\gamma_N^+\boldsymbol{u}$, however, is unknown. It is to be found by solving a boundary integral equation on Γ. The Boundary Element Method is an appropriate technique to solve this boundary integral equation numerically.

We would like to present a Boundary Element implementation using higher order $H(\mathrm{div}_\Gamma, \Gamma)$-conforming shape function. The code is related to an automatic *hp*-adaptive Finite Element implementation solving $2D$ Maxwell problems, [**?**]. Our idea has been to extend this code in order to solve $3D$ problems by an *hp*-Boundary Element Method. The power of this new implementation is the usage of the $H^1(\Gamma)$-conforming elements to describe curved surfaces and the $H(\mathrm{div}_\Gamma, \Gamma)$-conforming shape functions up to order nine. The degrees of freedom for higher order shape functions for the input data are determined by the so-called projection based interpolation. The discrete ansatz spaces conserve the exact sequence property.

A principal difference between the original Finite Element implementation and the Boundary Element implementation is the change of dimension. We are going to have a closer look at the Piola transform which is the crucial tool to construct conforming shape functions on curved elements in three space dimensions.

The numerical results show how the convergence improves for higher order discretisation.

References

[1] L. Demkowicz, *Computing with hp-adaptive finite elements. Vol. 1*, Chapman & Hall/CRC Applied Mathematics and Nonlinear Science Series (2007).